MATHEMATICS 10–20–30

A. PROGRAM RATIONALE AND PHILOSOPHY

To set goals and make informed choices, students need an array of thinking and problem-solving skills. Fundamental to this is an understanding of mathematical techniques and processes that will enable them to apply the basic skills necessary to address everyday mathematical situations, as well as acquire higher order skills in logical analysis and methods for making valid inferences.

A knowledge of mathematics is essential for a well-educated citizenry. However, the need for and use of mathematics in the life of the average citizen is changing. **Emphasis has shifted from the memorization of mathematical formulas and algorithms toward a more dynamic view of mathematics as a precise language, used to reason, interpret and explore.** There is still a need for the logical development of concepts and skills as a basis for the appropriate use of mathematical information to solve problems. The more traditional problem-solving techniques, combined with techniques such as estimation and simulation, ...rated with modern technology, are ...th which mathematical problems are

...the way in which mathematics is used ...ating a concurrent change in the ...of mathematics education. Students ...panded list of fundamental concepts ...so need to understand the ideas that ... those concepts and how they are related. They also require familiarity with their applications. Most important, students must be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge. In addition, the ability of technology to provide quick and accurate computation and manipulation, to enhance conceptual understanding and to facilitate higher order thinking, should be recognized and used by students.

The majority of students who enter senior high school exhibit mainly concrete operational behaviours with regard to mathematics. Students are expected to acquire much abstract understanding in senior high school mathematics courses. The course content of the Senior High School Mathematics Program is cognitively appropriate for the students and should be presented in a way that is consistent with the students' ability to understand.

The Senior High School Mathematics Program includes the course sequences Mathematics 16–26, 14–24, 13–23–33 and 10–20–30, plus Mathematics 31. Transfer by students among courses of different sequences is possible. The course sequences commensurate with differing abilities, interests and aspirations, are designed to enable students to have success in mathematics. As well, the mathematics program reflects the changing needs of society,

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Change in the way in which mathematics is used is necessitating a concurrent change in the emphases of mathematics education. Students need an expanded list of fundamental concepts but will also need to understand the ideas that make up those concepts and how they are related. They also require familiarity with their applications. Most important, students must be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge. In addition, the ability of technology to provide quick and accurate computation and manipulation, to enhance conceptual understanding and to facilitate higher order thinking, should be recognized and used by students.

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and provides students with the mathematical concepts, skills and attitudes necessary to cope with the challenges of the future.

The Mathematics 10–20–30 sequence is designed for students with an interest and aptitude in mathematics, who are intending to pursue post-secondary studies at a university or in a mathematics-intensive program at a technical school or college. Mathematics 10–20–30 emphasize the theoretical development of topics in algebra, geometry, trigonometry and statistics up to a level acceptable for entry into universities and other post-secondary institutions. Having successfully completed Mathematics 30, students will have fulfilled the mathematics requirement for the Advanced High School Diploma.

B. GENERAL LEARNER EXPECTATIONS

Students are expected to be mathematically literate at the conclusion of their senior high school mathematics education. **Mathematical literacy** refers to students' ability and inclination to manage the demands of their world through the use of mathematical concepts and procedures to communicate, reason and solve problems. More specifically, *students will be expected to:*

- have achieved understanding of the basic mathematical concepts, and developed the skills and attitudes needed to become responsible and contributing members of society
- apply basic mathematical skills and concepts in practical situations
- have developed the skills, concepts and attitudes that will ensure success in the mathematical situations that occur in future educational endeavours, employment and everyday life
- have developed the skills, concepts and attitudes that will enable the acquisition of mathematical knowledge beyond the conclusion of secondary education
- have developed critical and creative thinking skills
- be able to communicate mathematical ideas effectively
- understand how mathematics can be used to investigate, interpret and make decisions in human affairs
- understand how mathematics can be used in the analysis of natural phenomena
- understand the connections and interplay among various mathematical concepts and between mathematics and other disciplines
- understand and appreciate the positive contributions of mathematics, as a science and as an art, to civilization and culture.

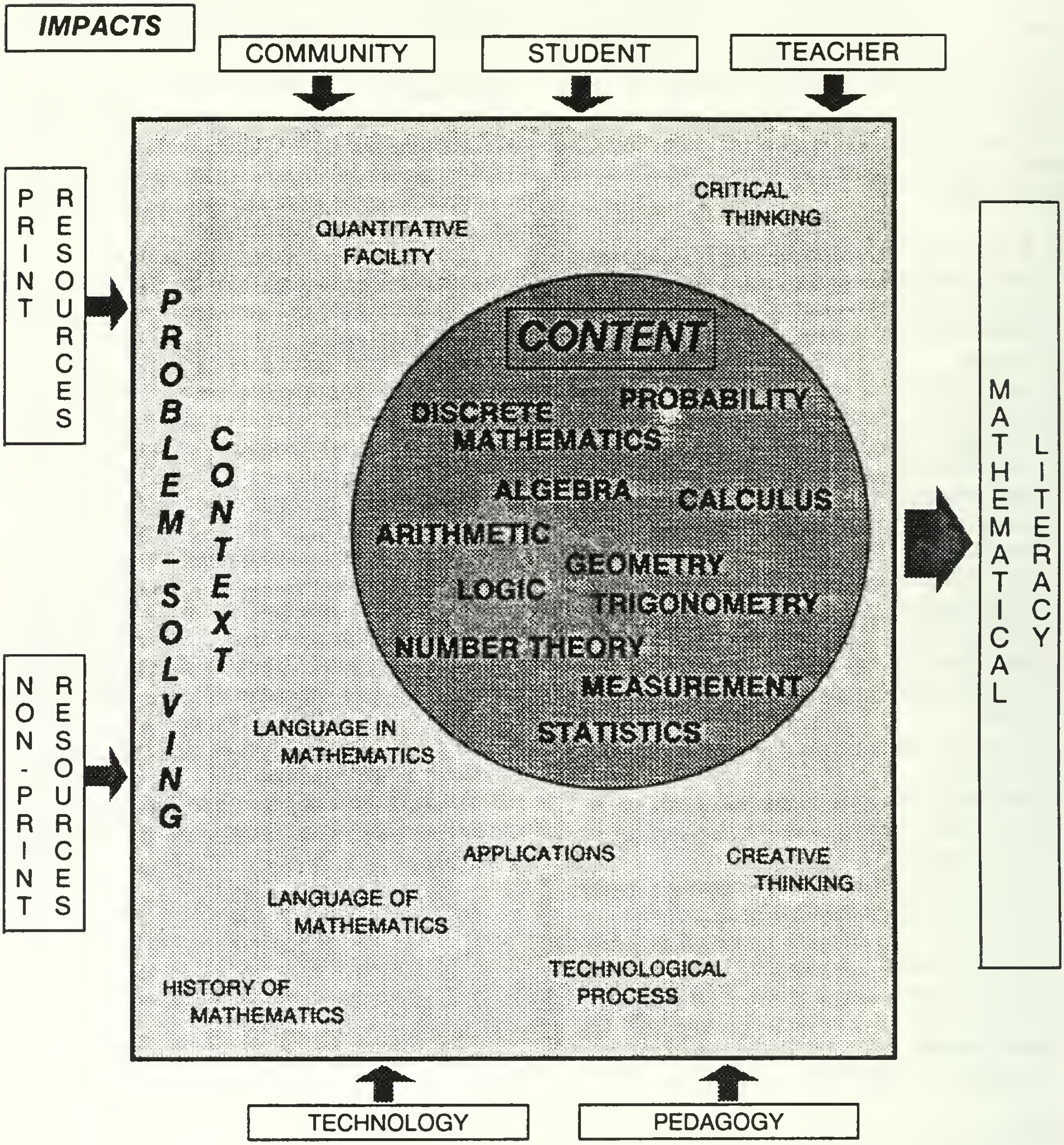
A General Model for Mathematical Literacy for Senior High School Programs, which outlines the factors that affect what and how students learn as they become mathematically literate, is presented on the following page. The model is fluid in that the content can be learned in a problem-solving context that engages any of a number of vehicles as the learning focus. At the same time, the students involved in the learning situation are affected in what and how they learn by forces that have impact on them.

The **Content** of the Senior High School Mathematics Program is the body of knowledge that is to be acquired by students. In the various senior high school mathematics courses, it is made up of topics that can be categorized into one or more of the listed strands.

In each course in the Senior High School Mathematics Program, students will focus on problem solving. The **Problem-Solving Context** refers to the instruction emphases within which the specific content expectations can be acquired. The various entries indicated in the model suggest processes that belong to the problem-solving context and may be used by students as vehicles for learning the content.

The **Impacts** on the problem-solving context are those skills, attitudes and experiences that are possessed by the students and teachers involved, as well as the resources they may use throughout the learning process. They include the influence exerted by the culture and beliefs of the community as reflected by the school. The effectiveness of the context in enabling a student to acquire the content is dependent on the skillful management by the teacher of those items that have an impact on a student's learning.

A General Model for Mathematical Literacy for Senior High School Programs



C. SPECIFIC LEARNER EXPECTATIONS

PROGRAM ORGANIZATION

The major part of the content of each senior high school mathematics course consists of topics required of all students who take the course. The **required content** comprises 80 per cent of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations for problem solving and the use of technology.

Each course includes a compulsory component comprising 20 per cent of the course, made up of **elective material** that is consistent with the content and expectations of the required component. **The elective material provides for enrichment, remediation, or innovative or experimental presentations or activities. It is not intended to provide acceleration or advanced placement.** However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Evaluation of students in the Senior High School Mathematics Program will involve assessment of the level of achievement of all of **the learner expectations, including concepts, skills and attitudes, as well as problem-solving and technological expectations.** For more information regarding evaluation, consult the *Teacher Resource Manual for Senior High Mathematics*.

PROGRAM STRUCTURE

At the beginning of each course is a list of **attitude** expectations. These attitudes embody a mathematical attitude or frame of mind for a student to view the world. The attitude expectations should be woven into the fabric of the entire course.

Following this are the **problem-solving** expectations that outline a variety of procedures, strategies, skills and checking techniques for solving problems. Because a major purpose for studying mathematics is to learn to solve problems, problem-solving expectations occur throughout all areas of the specific learner expectations. Students must have the background skills and knowledge necessary to achieve these expectations successfully, using problem-solving techniques.

The units of the course are broken into a number of concepts. Each unit begins with an **overview** of the concepts and skills included in the unit. This indicates reasons why the unit is being studied and how these particular concepts and skills fit into the development of major mathematical concepts.

A concept is an abstract or general idea about specific instances that have common properties or an identifiable relationship to one another. The concepts are presented as mathematical definitions or theorems or as statements of mathematical ideas or abstractions.

Supporting each concept are a number of skills. **Skills** are intellectual or physical capabilities that will be developed in the context of the particular concept.

Skills specifically related to the use of **technology** identify areas in which scientific calculators and/or computer technology are applied by students as tools to be used for calculations, manipulation or graphing, or to aid in the analysis of problems. Technological expectations are defined explicitly throughout the learner expectations. In many cases, a particular technology is indicated for investigation or analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving.

Students will be expected to learn how and when to use technology and have a demonstrated proficiency in **estimation** and **mental arithmetic**. To use technology effectively, they must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the result of a calculation.

The words verify and prove appear throughout the learner expectations. For the purposes of the Senior High School Mathematics Program, they are interpreted as:

- Verify – to substantiate the validity of an operation, solution, formula or theorem through the use of examples that may or may not be generalized;
- Prove – to substantiate the validity of an operation, solution, formula or theorem in general and to provide logical arguments for each step in the process.

Attitudes

1. *Students will be expected to* demonstrate an attitude associated with mathematical literacy. In particular, students will be expected to:

   1.1 be confident in their mathematical knowledge and in their ability to acquire new knowledge
   1.2 demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems
   1.3 develop intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge
   1.4 exhibit an attitude of curiosity and spontaneity, and appreciate creativity and innovation in representing situations mathematically
   1.5 be critical and constructive in approaching new ideas and new processes
   1.6 be aware of the importance of communication skills in mathematics
   1.7 appreciate the usefulness of computational competence, mathematical processes and problem-solving skills that are used in the decision-making and modelling processes in our society
   1.8 appreciate the contributions of mathematics to our culture and civilization.

Problem Solving

1. *Students will be expected to* demonstrate an understanding of the variety of procedures that can be used to understand problems. In particular, students will be expected to:

   1.1 read the problem thoroughly
   1.2 identify and clarify key components
   1.3 restate the problem, using familiar terms
   1.4 evaluate the given information as to sufficiency and relevancy
   1.5 interpret pictures, charts and graphs
   1.6 determine hidden assumptions
   1.7 ask relevant questions
   1.8 identify given, needed and wanted information
   1.9 diagram or model the problem situation
   1.10 use suitable notation
   1.11 determine valid inferences
   1.12 simulate a problem situation
   1.13 formulate situations into identifiable problems.

2. *Students will be expected to* develop a variety of strategies for use in the solution of mathematical problems. In particular, students will be expected to:

   2.1 conduct an investigation
   2.2 use estimation and approximation
   2.3 develop equations or use formulas
   2.4 use flow charts
   2.5 make lists and charts
   2.6 look for patterns
   2.7 work backward
   2.8 break the problem into smaller parts
   2.9 look for a simpler or related problem
   2.10 make diagrams or models
   2.11 use manipulatives
   2.12 choose and sequence a series of mathematical operations
   2.13 sketch the graph of a problem situation
   2.14 establish procedures to gather and organize data
   2.15 apply empirical or inductive processes
   2.16 use geometric construction and measurement techniques
   2.17 make and test a conjecture.

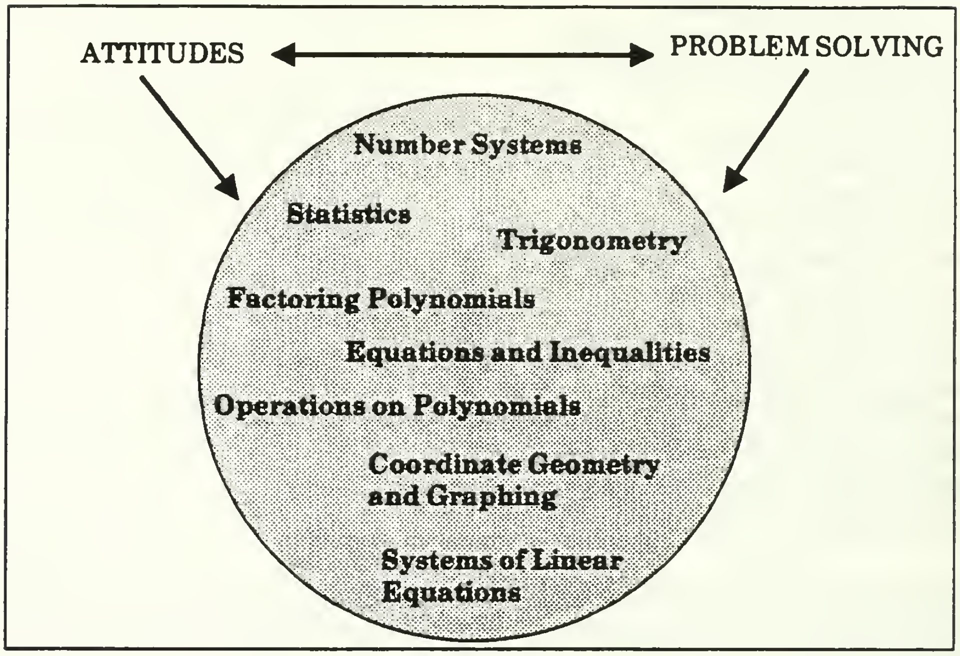
3. *Students will be expected to* develop a variety of skills that can be used to carry out the plan for the solution of a problem. In particular, students will be expected to:

   3.1 apply selected strategies
   3.2 present ideas clearly
   3.3 document the solution process
   3.4 use appropriate group behaviours
   3.5 use scientific graphing calculators and/or computers
   3.6 evaluate problem-solving strategies for effectiveness
   3.7 alter or abandon non-productive strategies
   3.8 search for additional information
   3.9 ask questions
   3.10 be open to inspirations, intuitions and "bright ideas".

4. *Students will be expected to* employ a variety of skills to help them look back over the solution of a problem. In particular, students will be expected to:

   4.1 determine the reasonableness of an answer
   4.2 explain the solution in oral or written form
   4.3 consider the possibility of additional solutions
   4.4 search for other strategies and processes of solution
   4.5 create and solve similar problems
   4.6 note the characteristics that will be identifiable in similar problems
   4.7 make a generalization
   4.8 examine the assumptions made and simplifications and modifications used for accuracy, effectiveness and efficiency.

MATHEMATICS 10 PROGRAM STRUCTURE



NUMBER SYSTEMS

1. *Students will be expected to* demonstrate an understanding that rational numbers are those which can be written as $\frac{a}{b}$, a, b$\in$I, b$\neq$0, and that they exhibit characteristics that are unique.

   Students will be expected to:

   1.1 identify repeating or terminating decimals as rational numbers
   1.2 change fractions to their equivalent decimal and per cent value
   1.3 change terminating decimals to fractions and per cent
   1.4 recognize the fractional equivalents of common repeating decimals (thirds, sixths, ninths).

2. *Students will be expected to* demonstrate an understanding that irrational numbers are those that cannot be written in the form $\frac{a}{b}$, a, b$\in$I, b$\neq$0.

   Students will be expected to:

   2.1 recognize irrational numbers as infinite, non-repeating decimals
   2.2 find approximations for square roots of non-perfect squares

      2.2.1 use scientific calculators to find approximations for the square roots of rational numbers

   2.3 recognize the square roots of negative integers as imaginary numbers
   2.4 recognize π as a special irrational number.

3. *Students will be expected to* demonstrate an understanding that real numbers are composed of rational and irrational numbers and that complex numbers are comprised of real and imaginary numbers.

Students will be expected to:

3.1 locate real numbers on a number line

3.1.1 represent rational and irrational numbers with measured and constructed line segments that correspond to the number

3.2 use absolute value to find the distance between points on a number line.

OPERATIONS ON POLYNOMIALS

1. *Students will be expected to* demonstrate an understanding that polynomials can be used to describe some mathematical situations in which values vary.

Students will be expected to:

1.1 identify the following as they occur in the study of polynomials: term, variable, factor, monomial, binomial, trinomial, polynomial, coefficient, degree, exponent, base, power
1.2 evaluate a polynomial for given values of the variables

1.2.1 write polynomials as rules which describe a mathematical situation.

2. *Students will be expected to* demonstrate an understanding that calculations involving expressions with exponents are subject to laws and properties.

Students will be expected to:

2.1 use the multiplication law of exponents for powers with literal bases and integral exponents, $a^m \times a^n = a^{m+n}$

2.1.1 verify the multiplication law of exponents

2.2 use the power law of exponents, $(a^m)^n = a^{mn}$, for expressions with literal bases and integral exponents

2.2.1 verify the power law of exponents

2.3 use the division law of exponents for powers with literal bases and integral exponents,

$$\frac{a^m}{a^n} = a^{m-n}$$

2.3.1 verify the division law of exponents.

3. *Students will be expected to* demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to:

3.1 add and subtract polynomials
3.2 multiply polynomials by monomials, and binomials by binomials
3.3 recognize the expansion of the square of a binomial and the product of conjugate binomials as special cases
3.4 divide polynomials by monomials

3.5 divide polynomials by binomials of the form $ax + b$, $a, b \in I$.

FACTORING POLYNOMIALS

1. *Students will be expected to* demonstrate an understanding that polynomials can be factored.

   Students will be expected to:

   1.1 factor polynomials that have a monomial common factor
   1.2 factor polynomials that have a binomial common factor
   1.3 factor trinomials of the form $ax^2 + bx + c$; $a, b, c \in I$
   1.4 factor perfect square trinomials
   1.5 factor difference of squares polynomials
   1.6 factor polynomials that require a combination of methods in order to be fully factored

   1.6.1 solve problems with factorable polynomials.

2. *Students will be expected to* demonstrate an understanding that the values of the variables which make a polynomial equal to zero can be found by factoring.

   Students will be expected to:

   2.1 find the rational zeros of an integral quadratic polynomial in one variable by factoring
   2.2 solve quadratic equations with integral coefficients in one variable by factoring

   2.2.1 solve problems by factoring which can be represented by quadratic equations.

EQUATIONS AND INEQUALITIES

1. *Students will be expected to* demonstrate an understanding that equations can be used to represent algebraically, situations and relationships that involve equalities.

   Students will be expected to:

   1.1 solve and verify the solution of any first degree equation in one variable by applying formal algebraic procedures

   1.1.1 represent problems with equations and solve the problems through the solution of the equations

   1.2 solve and verify the solution of simple quadratic and radical equations that can be reduced easily.

2. *Students will be expected to* demonstrate an understanding that some situations and relationships can be represented through the use of inequalities.

   Students will be expected to:

   2.1 solve, graph and verify the solution of any first degree inequality

   2.1.1 verify the "reverse the sign" rule in the inequalities
   2.1.2 represent problems with inequalities and solve the problems through the solution of the inequalities.

COORDINATE GEOMETRY AND GRAPHING

1. *Students will be expected to* demonstrate an understanding that points and lines can be represented on a Cartesian plane and that they have characteristics that can be defined and measured.

   Students will be expected to:

   1.1 use the formula for the distance between two points

      1.1.1 deduce the distance formula from the Pythagorean theorem
      1.1.2 solve problems that involve the use of the distance formula

   1.2 use the formula for the coordinates of the midpoint of a line segment

      1.2.1 verify the midpoint formula
      1.2.2 solve problems that involve the use of the midpoint formula

   1.3 use the formula for the slope of a line passing through two points

      1.3.1 solve problems that involve the use of formula for the slope of the segment between two points
      1.3.2 solve problems that involve the relationship between the slopes of parallel and perpendicular lines

   1.4 determine whether points in a plane are collinear

      1.4.1 solve problems that involve the necessary and sufficient conditions for collinearity.

2. *Students will be expected to* demonstrate an understanding that the ordered pairs that satisfy a linear equation, $Ax+By+C=0$, correspond to coordinates on the Cartesian plane and can be used to graph the equation on the Cartesian plane.

   Students will be expected to:

   2.1 find sets of ordered pairs that satisfy linear equations
   2.2 graph linear equations by plotting sets of ordered pairs that satisfy the equation
   2.3 find the x- and y-intercepts of linear equations
   2.4 graph linear equations by using the x- and y-intercepts
   2.5 write linear equations in the slope-intercept form, $y=mx+b$
   2.6 graph linear equations using the slope and the y-intercept or another point that satisfies the equation

      2.6.1 design and carry out an investigation involving the use of scientific calculators or computers to determine the effects of the parameters m and b on the graphs of linear equations

   2.7 identify and graph equations of lines parallel to the x and y axes.

3. *Students will be expected to* demonstrate an understanding that lines in a plane can be uniquely determined.

Students will be expected to:

3.1 graph and write the equation of a line, given the conditions that uniquely determine it

3.1.1 solve problems that involve conditions that uniquely determine a line.

SYSTEMS OF LINEAR EQUATIONS

1. *Students will be expected to* demonstrate an understanding that some mathematical circumstances can be represented algebraically by a system of two linear equations in two unknowns.

Students will be expected to:

1.1 solve simple systems of linear equations in two unknowns graphically
1.2 solve systems of linear equations in two unknowns algebraically

1.2.1 solve problems that can be represented with a system of two equations in two unknowns.

2. *Students will be expected to* demonstrate an understanding that lines in a plane can relate to each other in various ways.

Students will be expected to:

2.1 recognize the characteristics of systems of linear equations with graphs that are parallel, coincident or intersecting

2.2.1 verify the conditions that are present in a system of equations that determine the relationships of the graphs.

STATISTICS

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1. *Students will be expected to* demonstrate an understanding that populations have certain characteristics which can be determined by analyzing the characteristics of a suitable sample of that population.

   Students will be expected to:

   1.1 determine a sample that will adequately and accurately represent a population
   1.2 evaluate samples for bias, appropriateness of sample type, amount of data and randomness
   1.3 draw inferences about the population from which a sample was taken and express it using the language of probability

   1.3.1 determine a valid sample, design and carry out a simple survey, and make a valid inference to the population based on the results
   1.3.2 defend or oppose a generalization that is made from a sample to a population.

2. *Students will be expected to* demonstrate an understanding that data collected from a sample must be organized, presented and analyzed in order that valid inferences will be drawn.

   Students will be expected to:

   2.1 identify a situation, formulate an hypothesis and plan what data need to be collected
   2.2 organize data using stem-and-leaf plots and box plots
   2.3 choose and calculate appropriate measures of central tendency for sets of data
   2.4 represent data using appropriate graphs
   2.5 do simple visual analyses on sets of data organized in stem-and-leaf plots, box plots and graphs

   2.5.1 derive and defend conclusions based on the visual analyses of sets of data

   2.6 evaluate intuitively an analysis of data for the confidence with which the results can be accepted.

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Learner Expectations

1. *Students will be expected to* demonstrate an understanding that a bivariate distribution involves two variables that may have some relationship to each other.

   Students will be expected to:

   1.1 plot sets of bivariate data on a scatterplot
   1.2 plot a line of best fit on a scatterplot, using eye and the median fit method
   1.3 develop and use prediction equations of the line of best fit to make inferences for populations
   1.4 recognize and describe, qualitatively, the apparent correlation between the variables of a bivariate distribution from a scatterplot
   1.5 collect, organize and analyze sets of bivariate data

   1.5.1 apply statistical processes and statistical reasoning in investigations involving bivariate data.

TRIGONOMETRY

1. *Students will be expected to* demonstrate an understanding that similar triangles have equal angles and that the corresponding sides are proportional.

   Students will be expected to:

   1.1 recognize and write the similarity relationship between similar triangles
   1.2 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides

      1.2.1 generalize the properties of similar triangles through the use of inductive processes
      1.2.2 solve problems involving the use of the properties of similar triangles
      1.2.3 develop the sine, cosine and tangent ratios in similar right triangles through the use of the properties of similar triangles.

2. *Students will be expected to* demonstrate an understanding that the sine, cosine and tangent ratios of a given angle are particular ratios between pairs of corresponding sides of similar, right triangles that contain the given angle.

   Students will be expected to:

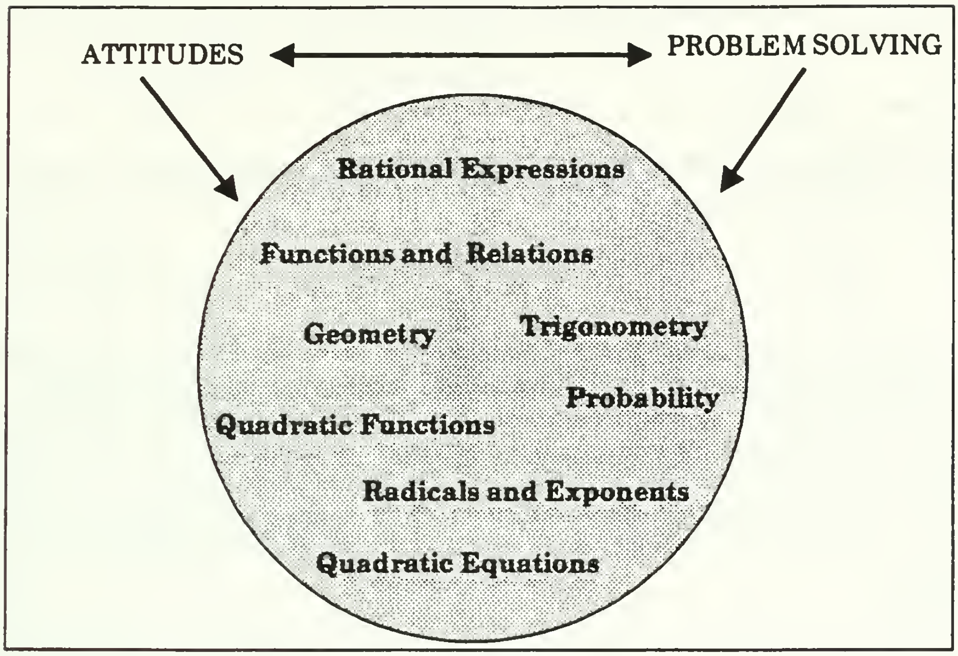
   2.1 find the sine, cosine and tangent ratios, and the measures of the acute angles within right triangles, given the measures of any two sides

      2.1.1 use a calculator to find the sine, cosine and tangent ratios of any acute angle
      2.1.2 use a calculator to find the measure of an acute angle given the value of one of the trigonometric ratios of the angle
      2.1.3 solve problems in which an angle measure in a right triangle must be found

   2.2 find the measure of a side of a right triangle given the measure of a second angle and the length of a side

      2.2.1 solve problems in which the length of a side of a right triangle must be found.

MATHEMATICS 20 PROGRAM STRUCTURE



RADICALS AND EXPONENTS

1. *Students will be expected to* demonstrate an understanding of the relationship between radical expressions and exponential expressions with rational exponents and an understanding that these expressions are subject to the laws of exponents.

   Students will be expected to:

   1.1 transform expressions from radical to exponential form and vice versa
   1.2 simplify expressions that are written in radical or exponential form

   1.2.1 evaluate radical or exponential expressions, using a calculator.

2. *Students will be expected to* demonstrate an understanding that radical expressions and equations that contain radicals can be evaluated and simplified by performing basic arithmetic operations.

   Students will be expected to:

   2.1 change the form of square root expressions from mixed to entire radicals and vice versa
   2.2 perform the operations of addition, subtraction and multiplication involving square root expressions
   2.3 solve radical equations

   2.3.1 solve problems involving radical equations.

RATIONAL EXPRESSIONS

1. *Students will be expected to* demonstrate an understanding that rational expressions are fractions in which the numerators and denominators are polynomials, and that operations can be performed on such expressions.

   Students will be expected to:

   1.1 determine the non-permissible replacement values for the variable in rational expressions in one variable

      1.1.1 use a calculator or computer to graph rational functions to determine non-permissible values

   1.2 simplify rational expressions by factoring
   1.3 perform the operations of multiplication and division on rational expressions
   1.4 perform the operations of addition and subtraction involving two rational expressions
   1.5 solve equations involving rational expressions

      1.5.1 solve problems that can be represented by equations containing rational expressions.

PROBABILITY

1. *Students will be expected to* demonstrate an understanding that probability describes the likelihood of the occurrence of an event and is a number between 0 and 1.

   Students will be expected to:

   1.1 use the language of probability
   1.2 determine the sample space for simple events
   1.3 determine the probability of simple events through experiments

      1.3.1 carry out an investigation to determine the experimental probability of an event

   1.4 determine the theoretical probability of events that have easily countable sample spaces

      1.4.1 compare theoretical probabilities to those for the same event generated empirically.

2. *Students will be expected to* demonstrate an understanding that the probability of a compound event can be determined from the probability of the individual events.

   Students will be expected to:

   2.1 find the probability of two or more events occurring together by the application of the multiplication law for independent events $P(A \text{ and } B) = P(A) \times P(B)$ and for dependent events $P(A \text{ and } B) = P(A) \times P(B|A)$

      2.1.1 solve problems that involve finding the probability of the occurrence of two or more events
      2.1.2 determine experimentally the probability of two or more events occurring together and compare it to the theoretical probability

2.2 find the probability of the occurrence of one or the other of two events, A and B, by the application of the addition law $P(A \text{ or } B) = P(A)+P(B)$ for mutually exclusive events and, in general, $P(A \text{ or } B) = P(A)+P(B)-P(A \text{ and } B)$

2.2.1 determine experimentally the probability of the occurrence of events A or B and compare it to the theoretical probability
2.2.2 solve problems that involve finding the probability of either of two events.

3. *Students will be expected to* demonstrate an understanding that simulations are experiments that represent the conditions present in real situations through the use of devices and processes with known probabilities.

Students will be expected to:

3.1 design and carry out simulations involving events that have known probabilities

3.1.1 use simulation models to solve problems involving events with known probabilities

3.2 design and carry out simulations involving events that have unknown probabilities

3.2.1 use simulation models to solve problems involving events with unknown probabilities.

FUNCTIONS AND RELATIONS

1. *Students will be expected to* demonstrate an understanding that certain observed real-world phenomena are quantitatively related to each other and that these relations can be described graphically, with sets of ordered pairs, rules and equations.

Students will be expected to:

1.1 graph relations that describe physical phenomena or everyday situations

1.1.1 solve problems by graphing and interpreting the graphs that describe physical phenomena and everyday occurrences

1.2 determine the domain and the range of relations algebraically and from given graphs.

2. *Students will be expected to* demonstrate an understanding that for some relations, called functions, the value of the independent variable (domain) uniquely determines the value of the function (range, dependent variable).

Students will be expected to:

2.1 represent mathematical situations such as direct, inverse and partial variations with tables of values, identify the dependent and independent variables, and express the domain and range, appropriately noting any restrictions
2.2 interpolate and extrapolate from the graphs of functional relationships
2.3 use functional notation and graphs to describe functional relationships

2.3.1 solve problems algebraically or by the use and interpretation of graphs that represent functions

2.4 determine those relations that are functions

2.4.1 develop and explain tests that could be used to determine if any relation is or is not a function

2.5 illustrate and recognize different kinds of functions algebraically and graphically from the following list: linear functions (including identity and constant functions), polynomial functions (including quadratic and cubic functions), reciprocal functions, absolute value functions and exponential functions

2.5.1 draw and analyze the graphs of functions, using calculators or computers

2.6 identify the zeros of a function as the x-intercepts of its graph

2.6.1 find the zeros of a function by analyzing its graph and its value for various replacements of the independent variable, using calculators or computers

2.7 write, and sketch the graphs of, the inverses of relations and functions.

3. *Students will be expected to* demonstrate an understanding of how particular parameters can be used to effect translations, reflections or vertical stretchings of the graph of any function.

Students will be expected to:

3.1 describe the transformation effect on the graph of $y=f(x)$ of the parameters a and b in $y=f(x-a)+b$

3.1.1 perform an investigation to determine the effect of the parameters a and b on the graph of $y=f(x-a)+b$

3.2 describe the transformation effects on the graph of $y=f(x)$ of the parameter c in $y=cf(x)$

3.2.1 perform an investigation to determine the effects of the parameter c on the graph of $y=cf(x)$

3.3 describe and sketch the graphs of $y=cf(x-a)+b$ by applying the transformation effects of a, b and c on the graph of $y=f(x)$

3.3.1 predict the graphs of functions written in the form $y=cf(x-a)+b$ given the graph of $y=f(x)$, and verify, using calculators or computers.

QUADRATIC FUNCTIONS

1. *Students will be expected to* demonstrate an understanding that any function that can be written in the form $y=ax^2+bx+c$, $a\neq 0$, is a quadratic function and that individual quadratic functions have unique characteristics and graphs.

Students will be expected to:

1.1 sketch the graphs of quadratic functions written in standard form $y=a(x-h)^2+k$

1.1.1 investigate the effects of the parameters a, h and k in $y=a(x-h)^2+k$, using a calculator or computer

1.2 transform quadratic functions from the general form $y=ax^2+bx+c$ to the standard form $y=a(x-h)^2+k$ by completing the square

1.3 find the vertex, axis of symmetry, domain, range, maximum or minimum values and x- and y-intercepts of a quadratic function from its equation or from its graph

1.3.1 solve problems that involve quadratic functions, by analyzing the functions depicted in graphical and equation form.

QUADRATIC EQUATIONS

1. *Students will be expected to* demonstrate an understanding that finding the x-intercepts of the quadratic function $y = ax^2 + bx + c$ is equivalent to solving the quadratic equation $ax^2 + bx + c = 0$.

Students will be expected to:

1.1 use the quadratic formula to solve quadratic equations

1.1.1 solve problems that involve quadratic equations

1.2 solve equations that contain radicals and rational expressions

1.2.1 solve problems that involve equations containing radicals and rational expressions

1.3 evaluate the discriminant of a quadratic equation and state the nature of its roots
1.4 recognize and explain the relationship between the graphs of quadratic functions and the real or non-real nature of the roots of their corresponding equations.

GEOMETRY

1. *Students will be expected to* demonstrate an understanding that the perpendicular bisector of a chord passes through the centre of the circle, and that this relationship can be expressed and verified in different ways.

Students will be expected to:

1.1 demonstrate, by construction or by computer simulation, that the perpendicular bisector of a chord passes through the centre of the circle
1.2 verify the relationship amongst chords, their perpendicular bisectors and the centres of circles, using analytic geometry

1.2.1 solve problems that involve the relationship amongst chords, their perpendicular bisectors and the centres of circles
1.2.2 calculate the length of a chord, the distance from a chord to the centre of the circle and the radius of the circle given any two of the measurements

1.3 provide a logical argument to support the relationship amongst chords, their perpendicular bisectors and the centres of circles.

2. *Students will be expected to* demonstrate an understanding that if a line is tangent to a circle, then it is perpendicular to a radius drawn to the point of contact, and that this relationship can be expressed and verified in different ways.

Students will be expected to:

2.1 demonstrate, by construction or by computer simulation, that a perpendicular drawn from the point of contact of a tangent passes through the centre of the circle

2.2 verify the relationship between a tangent and the radius drawn to the point of contact, using analytic geometry

   2.2.1 solve problems that deal with the relationship between a tangent and the radius drawn to the point of contact
   2.2.2 calculate, given a point in the exterior of a circle, the length of tangent segments from the point, the lengths of radii to the points of contact of the tangents, and the distance from the point to the centre of the circle, given any two of the measurements

2.3 provide a logical argument to support the relationship between a tangent and the radius drawn to the point of contact.

TRIGONOMETRY

1. *Students will be expected to* demonstrate an understanding that angles can be drawn on a coordinate plane and that their trigonometric ratios can be determined with respect to the coordinates of points on the plane.

   Students will be expected to:

   1.1 recognize and sketch positive and negative angles in standard position
   1.2 determine the reference angle for an angle drawn in standard position on a coordinate plane
   1.3 express the sine, cosine and tangent ratios for any angle, θ, in standard position on a coordinate plane in terms of x, y and r, where r is the length of the terminal side and (x, y) is the end point
   1.4 determine the sine, cosine and tangent ratios for any angle

      1.4.1 use a calculator to determine the sine, cosine and tangent ratios of any angle

   1.5 determine any two values of x, y, r and θ for an angle in standard position, given the other two.

2. *Students will be expected to* demonstrate an understanding that the methods used in the solution of right triangles can be used to develop laws for use in the solution of oblique triangles.

   Students will be expected to:

   2.1 find the measures of sides and angles in diagrams involving multiple right triangles in two or three dimensions

      2.1.1 solve problems involving multiple right triangles in two or three dimensions

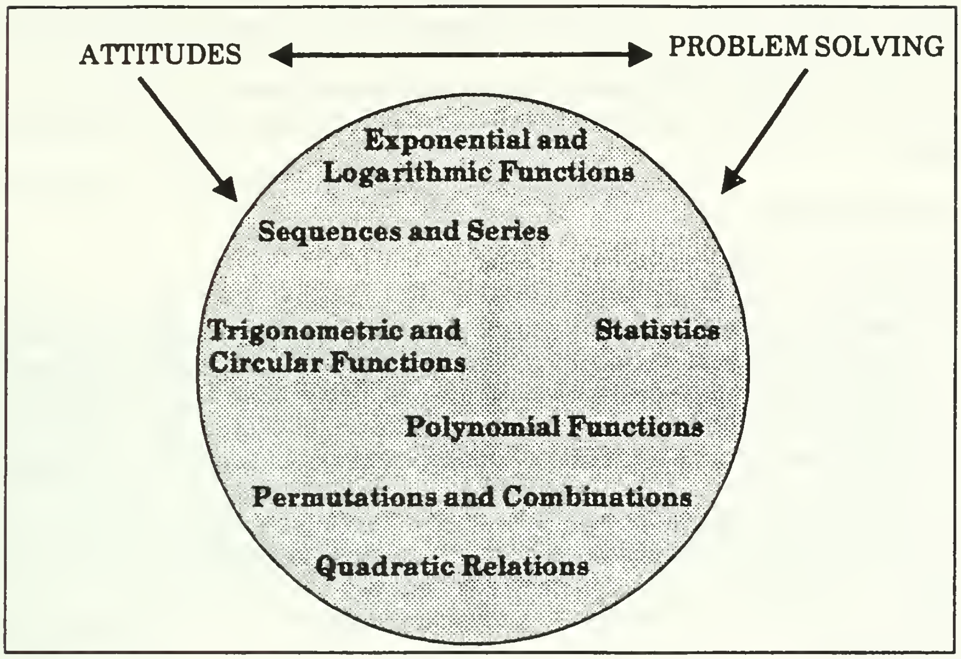
   2.2 find the measures of unknown sides and angles in oblique triangles by applying the sine law

      2.2.1 verify the sine law
      2.2.2 solve problems, including the ambiguous case, by using the sine law

   2.3 find the measures of unknown sides and angles in oblique triangles by applying the cosine law

      2.3.1 verify the cosine law
      2.3.2 solve problems by using the cosine law
      2.3.3 recognize that the cosine law is the generalization of the Pythagorean theorem.

MATHEMATICS 30 PROGRAM STRUCTURE



POLYNOMIAL FUNCTIONS

Overview

Much of the study of high school algebra is to provide the student with the ability to analyze general functions. For most high school students the study of polynomial functions is in many ways the culmination of their study of algebra. They should understand that algebra provides a means of operating with concepts at an abstract level and a process that can foster generalizations and insights beyond the original context.

The students' study of polynomial functions should not focus on developing mere manipulative facility but, rather, should emphasize conceptual understanding, coming to understand algebra as a means of representing general cases and a problem-solving tool. Emphasis should be on interpreting the graphs of functions, exploring the properties of graphs and determining how these properties relate to the forms of the corresponding equations.

Students should be aware that polynomial functions are very useful for describing relations among variables in a vast array of real-world situations. For example, analysis of polynomial functions frequently arises in the management sciences for examining cost, revenue and profit in the production and sale of goods. The use of computer technology and the methods associated with this technology, combined with a conceptual understanding of polynomial functions, enables students to gain powerful techniques for the analysis of complex functions.

The analysis of polynomial functions is one of the underpinnings to the study of calculus. The students' ability to visualize the graph of the function and to find the zero of the function are some of the ingredients in understanding calculus.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that a polynomial function is a function of the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0$ where $a_0, a_1 \ldots a_n$ are real numbers and $n \in N$.

2. *Students will be expected to* demonstrate an understanding that a polynomial function can be graphed on a Cartesian plane and that such graphs will have particular characteristics depending on the function.

   Students will be expected to:

   2.1 sketch the graphs of integral polynomial functions

      2.1.1 draw the graphs of integral polynomial functions using calculators or computers
      2.1.2 investigate the characteristics of the graphs of polynomial functions of different degrees and determine the effects of a multiplicity of zeros on the graphs of polynomial functions
      2.1.3 find approximations for the zeros of integral polynomial functions using calculators or computers
      2.1.4 analyze points on the graphs of polynomial functions using calculators or computers
      2.1.5 solve problems that can be represented by polynomial functions.

3. *Students will be expected to* demonstrate an understanding that many polynomial functions can have the same zeros.

   Students will be expected to:

   3.1 derive an equation of an integral polynomial function given its zeros
   3.2 derive the equation of an integral polynomial function given its zeros and an ordered pair that satisfies it

      3.2.1 find the equation of a polynomial function given its zeros and any other information that will uniquely define it.

4. *Students will be expected to* demonstrate an understanding of the following form of the division algorithm for polynomials: If any polynomial $P(x)$ is divided by a binomial of the form $(x-a)$ (called $D(x)$), the result will be a polynomial quotient $Q(x)$ and a remainder R.

   Students will be expected to:

   4.1 divide integral polynomial functions in one variable by a binomial
   4.2 write the division operation on a polynomial function by a binomial in the form of the Division Algorithm: $P(x) = D(x)Q(X) + R$.

5. *Students will be expected to* demonstrate an understanding that when a polynomial $P(x)$ is divided by a binomial of the form $(x-a)$, the remainder R is equal to $P(a)$ (Remainder Theorem).

   Students will be expected to:

   5.1 use the Remainder Theorem to evaluate polynomial functions for rational values of the variable

      5.1.1 prove the Remainder Theorem
      5.1.2 use the Remainder Theorem to prove that if a number a is a zero of a polynomial function $P(x)$ then $(x-a)$ will be a factor of $P(x)$ (Factor Theorem)

5.2 use the Factor Theorem to factor an integral polynomial function completely and to determine all of its real zeros

5.2.1 use a technology to factor polynomial functions

5.2.2 recognize that all rational zeros of a polynomial function will be of the form p/q, where p is a factor of a_0 and q is a factor of a_n.

TRIGONOMETRIC AND CIRCULAR FUNCTIONS

Overview

Trigonometry is not only an important and powerful tool for science and engineering, but is also esthetically attractive for many students because of its regularity and symmetry. The study of trigonometry in Mathematics 30 builds on the understanding of trigonometric ratios and how to use these ratios to solve real-world problems, and on the study of functions. In Mathematics 30, the ratios of right-angle trigonometry are generalized to both trigonometric and circular functions. Understanding of these functions and the connections between geometry and algebra are important in the future development of such topics as matrix representations of rotations, direction angles of vectors, polar coordinates and trigonometric representations of complex numbers. The trigonometric and circular functions are mathematical models for many periodic real-world phenomena, such as uniform circular motion, temperature changes, biorhythms, sound waves and tide variation.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that the radian measure of an angle is the ratio of the arc it subtends to the radius of a circle in which it is a central angle, and that one radian is the measure of a central angle subtended in a circle by an arc whose length is equal to the radius of the circle.

   Students will be expected to:

   1.1 identify the radian measure of a central angle in a circle

   1.2 convert angle measurements between degree and radian measure and vice versa

   1.3 determine the exact values of the trigonometric ratios for angles coterminal with $\frac{n\pi}{6}$, $\frac{n\pi}{4}$, $\frac{n\pi}{3}$, $\frac{n\pi}{2}$, and $n\pi$, $n \in I$.

2. *Students will be expected to* demonstrate an understanding that identities are statements of equality that are true for all values of the variable and that trigonometric identities are equations that express relations among trigonometric functions that are valid for all values of the variables for which the functions are defined.

   Students will be expected to:

   2.1 use the following fundamental trigonometric identities:

Reciprocal Identities

$$\frac{1}{\sin a} = \csc a$$

$$\frac{1}{\cos a} = \sec a$$

$$\frac{1}{\tan a} = \cot a$$

Quotient Identities

$$\frac{\sin a}{\cos a} = \tan a$$

$$\frac{\cos a}{\sin a} = \cot a$$

Pythagorean Identities

$$\sin^2 a + \cos^2 a = 1$$

$$\tan^2 a + 1 = \sec^2 a$$

$$\cot^2 a + 1 = \csc^2 a$$

2.1.1 derive the quotient and Pythagorean identities using logical processes

2.1.2 use the fundamental trigonometric identities to simplify, evaluate and prove trigonometric expressions involving identities

2.2 use the addition and subtraction identities (formulas):

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$.

3. *Students will be expected to* demonstrate an understanding that trigonometric functions can be graphed on a Cartesian plane.

Students will be expected to:

3.1 graph the following forms of the sine, cosine and tangent functions:

$y = a \sin[b(\theta + c)] + d$

$y = a \cos[b(\theta + c)] + d$

$y = \tan \theta$

3.1.1 use calculators or computers to draw and analyze the graphs of trigonometric functions

3.1.2 investigate the effects of the parameters a, b, c and d on the graphs of trigonometric functions using calculators or computers

3.1.3 state the domain and range of all the trigonometric functions.

4. *Students will be expected to* demonstrate an understanding of the methods used to solve trigonometric equations.

Students will be expected to:

4.1 solve first and second degree trigonometric equations involving multiples of angles on the domain $0 \leq \theta < 2\pi$

4.1.1 use calculators or computers to solve trigonometric equations by evaluating the graphs of trigonometric functions.

4.2 demonstrate the relationship between the root of a trigonometric equation and the graph of the corresponding function.

Delete Effective September 1, 1994

Overview

Statistics forms an essential part of the modern mathematics curriculum. Today, information is the most important commodity in our society. This daily barrage of information must not only be understood but used by students. Statistics begins with a problem that needs to be solved. It includes all the background thinking that occurs long before any work is done.

The techniques and skills used include the representation of data by line plots, box plots and calculating various averages. This concept involves situations where two variables are measured for each individual in a group. The resulting graph, called a scatterplot, is a useful technique for investigating the relationship between two variables. The line and equation can be used to make predictions. Surveys are an integral part of inferential statistics and a student must demonstrate an understanding that the result of a survey can be interpreted with measurable degrees of confidence. Knowledge of this type of statistics is necessary if students are to become intelligent consumers who can make critical and informed decisions.

Insert Effective September 1, 1994

Overview

One form of data that is often encountered is that which has a normal distribution. Normally distributed data has particular interest to statisticians who wish to make predictions about a population based upon known data.

Learner Expectations

Delete Effective September 1, 1994
(Moved to Mathematics 10
Effective September 1, 1996)

1. *Students will be expected to* demonstrate an understanding that a bivariate distribution involves two variables that may have some relationship to each other.

   Students will be expected to:

   1.1 plot sets of bivariate data on a scatterplot
   1.2 plot a line of best fit on a scatterplot using the median fit method
   1.3 develop and use prediction equations of the line of best fit to make inferences for populations
   1.4 recognize and describe the apparent correlation between the variables of a bivariate distribution from a scatterplot
   1.5 collect, organize and analyze sets of bivariate data

      1.5.1 apply statistical processes and statistical reasoning in investigations involving bivariate data.

2. *Students will be expected to* demonstrate an understanding that data can be distributed normally, and that a normal distribution has particular characteristics that can be used to describe and analyze many situations.

Students will be expected to:

2.1 find and interpret the mean and standard deviation of a set of normally distributed data

2.1.1 use calculators or computers to calculate the mean and standard deviation of sets of normally distributed data

2.2 apply the characteristics of a normal distribution

2.2.1 solve problems involving data that are normally distributed

2.3 find and apply the standard normal curve and the z-scores of data that are normally distributed

2.3.1 apply z-scores to solve problems involving probability distributions.

Delete Effective September 1, 1994
(No Replacement)

3. *Students will be expected to* demonstrate an understanding that the results of a survey can be interpreted with measurable degrees of confidence.

Students will be expected to:

3.1 distinguish between a population and a sample and assess the strengths, weaknesses and biases of given samples

3.2 collect and organize the results of yes/no surveys taken from defined samples

3.2.1 design and administer a simple survey

3.2.2 collect and organize the results of a simple survey

3.3 draw box plots of the results of multiple samples

3.3.1 carry out investigations involving multiple samples taken from populations with known and unknown proportions of yes responses

3.4 use charts of 90 per cent box plots to find the confidence interval within which a survey result can be interpreted

3.4.1 use statistical inferences to solve problems

3.5 draw statistical conclusions, make inferences to populations and explain the confidence with which such conclusions and inferences are made based on the results of yes/no surveys

3.5.1 design and administer a survey to a random sample of a population, collect and organize the responses, and analyze the results, including making inferences to the population and evaluating the results for the confidence with which they may be held.

QUADRATIC RELATIONS

Overview

The study of quadratic relations connects several topics from the high school mathematics program. The topics of coordinate geometry and algebra are linked to the analysis of relations and the resulting graphs of the locus of these relations. Because there are a number of practical phenomena that can be described in terms of quadratic relations it is possible for students to see the connection between real-world applications and theoretical representations. Since all conic sections can be defined as the locus of a point such that the ratio of its distance from a fixed point and a fixed line is constant and each of the four conics can be generated by altering this ratio, the study of quadratic relations provides the student with the opportunity to analyze a complex mathematical relation to determine common properties and structures.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding of the physical properties of the conic sections with respect to the intersection of a plane and a cone.

   Students will be expected to:

   1.1 describe the conic section formed by the intersection of a plane and a cone

   1.1.1 identify the point at which each of the conics becomes degenerate.

Delete Effective September 1, 1994

2. *Students will be expected to* demonstrate an understanding of the general quadratic relation $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ as the algebraic representation of any conic.

Insert Effective September 1, 1994

2. *Students will be expected to* demonstrate an understanding of the general quadratic relation $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ where $B=0$ as the algebraic representation of any conic section with an axis of symmetry parallel to one of the coordinate axes.

   Students will be expected to:

   2.1 describe the conics that would be generated by various combinations of values for the numerical coefficients

   2.1.1 investigate and describe the effects of the numerical coefficients on the graphs of quadratic relations, using calculators or computers.

3. *Students will be expected to* demonstrate an understanding of the effects of the numerical coefficients in the general quadratic relation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where $B = 0$ on the curves of the resulting conics.

   Students will be expected to:

   3.1 analyze the graphs of ellipses, parabolas and hyperbolas, given their equations

   3.1.1 use calculators or computers to draw the graphs of ellipses, parabolas and hyperbolas
   3.1.2 recognize which conditions are required for an ellipse to become a circle
   3.1.3 investigate and describe the effects of the numerical coefficient on the orientation, size and shape of the graph.

4. *Students will be expected to* demonstrate an understanding that a locus is a system of points that satisfies a given condition.

   Students will be expected to:

   4.1 recognize that each conic can be described as a locus of points

   4.1.1 use the locus definition to verify the equations that describe the conics
   4.1.2 solve problems that involve analyzing and determining the characteristics of a body that follows a conical path
   4.1.3 solve problems that involve analyzing and determining the characteristics of a conical surface.

5. *Students will be expected to* demonstrate an understanding that any conic can be described as the locus of points, such that the ratio of the distance between any point and a fixed point to the distance between the same point and a fixed line is a constant. This ratio is called eccentricity.

EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Overview

The study of exponential and logarithmic functions is an application of the general study of functions from Mathematics 20. For example, the logarithmic and exponential functions provide a good example of inverse functions. In this unit of study, students have further opportunity to understand the connections of functions to real-world phenomena, by using certain phenomena as models of functions in symbolic form and then graphing the functions. The study of the operations in logarithms and logarithmic equations provides the student with the opportunity to understand how a mathematics system operates and realize that the rules that govern properties and solution of equations apply to the system as a whole.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that an exponential function is one in which the variable appears in the exponent.

   Students will be expected to:

   1.1 sketch the graph of exponential functions of the form $y = a^x, a > 0$
   1.2 use the graphs of exponential functions to estimate the values of roots and powers

      1.2.1 draw and analyze the graphs of exponential functions using calculators or computers
      1.2.2 determine the domain and range of the exponential functions

   1.3 solve and verify exponential equations.

2. *Students will be expected to* demonstrate an understanding that many real-world phenomena exhibit exponential properties.

   Students will be expected to:

   2.1 recognize exponential functions describing situations involving exponential growth and decay

      2.1.1 solve problems involving exponential growth and decay.

3. *Students will be expected to* demonstrate an understanding of the characteristics and applications of logarithmic functions.

   Students will be expected to:

   3.1 draw the graphs of logarithmic functions as the inverses of exponential functions
   3.2 use the graphs of logarithmic functions to find the values of one of the variables, given the other variable

      3.2.1 draw and analyze the graphs of logarithmic functions using calculators or computers
      3.2.2 determine the domain and range of the logarithmic functions

   3.3 convert functions from exponential form to logarithmic form and vice versa.

4. *Students will be expected to* demonstrate an understanding that operations with logarithms are subject to basic properties and laws.

   Students will be expected to:

   4.1 apply the following laws and properties of logarithms:
   $\log_a mn = \log_a m + \log_a n$
   $\log_a \frac{m}{n} = \log_a m - \log_a n$
   $\log_a m^n = n\log_a m$

      4.1.1 evaluate logarithmic expressions using calculators and computers

   4.2 solve and verify logarithmic equations

      4.2.1 solve and verify logarithmic equations using calculators or computers.

5. *Students will be expected to* demonstrate an understanding that a logarithm with a base of 10 is a common logarithm.

   Students will be expected to:

   5.1 solve logarithmic equations and evaluate logarithmic expressions using common logarithms.

6. *Students will be expected to* demonstrate an understanding that many phenomena exhibit characteristics that can be described using logarithmic functions.

   Students will be expected to:

   6.1 recognize logarithmic functions that describe situations that have logarithmic characteristics

      6.1.1 solve problems that exhibit logarithmic properties by developing and solving logarithmic equations.

PERMUTATIONS AND COMBINATIONS

Overview

An understanding of permutations and combinations is becoming increasingly important for processing, analyzing and communicating information. One example of the use of permutations is in determining security codes for computers. The ability of computers to try large numbers of different permutations of numerals rapidly makes the design of computer security systems much more difficult. Whole new branches of mathematics are being developed to deal with problems of this nature. Understanding of permutations and combinations is important to building an understanding of formal concepts of theoretical probability and to be able to interpret and judge the validity of statistical claims in view of underlying probabilistic assumptions. In determining the probability of an event, the number of ways of an event happening must be compared to the total number of possible outcomes. This counting of alternatives cannot always be easily determined by simple enumeration. The study of permutations and combinations provides methods for counting under complicated conditions.

Emphasis in this unit is on the development of students' ability to solve problems, using combinatorial analysis as opposed to the simple application of analytic formulas for permutations and combinations. In Mathematics 20, students learned about probability and how to find the probability that two events would occur. Students in Mathematics 20 were also expected to be able to design and carry out simulations.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding of the Fundamental Counting Principle.

   Students will be expected to:

   1.1 calculate the total number of ways that a multiple of tasks can be conducted if each task can be performed in a multiple of ways

      1.1.1 solve problems that involve the use of the fundamental counting principle.

2. *Students will be expected to* demonstrate an understanding that a permutation is an arrangement in which the order is important.

Students will be expected to:

2.1 calculate the number of permutations there are of n things taken r at a time by applying the following formula: ${}_nP_r = \dfrac{n!}{(n-r)!}$

2.1.1 calculate the ${}_nP_r$ using calculators and computers
2.1.2 solve problems involving linear permutations, permutations with repetitions, circular and ring permutations
2.1.3 solve probability questions that involve the use of permutations.

3. *Students will be expected to* demonstrate an understanding that a combination is an arrangement in which the order is not important.

Students will be expected to:

3.1 calculate the number of combinations there are of n things taken r at a time by applying the following formula: ${}_nC_r = \dfrac{n!}{r!(n-r)!}$

3.1.1 calculate ${}_nC_r$ using a calculator or computer
3.1.2 solve problems including probability problems that involve the use of combinations.

4. *Students will be expected to* demonstrate an understanding that the numerical coefficients of the terms in a binomial expansion can be determined using the Binomial Theorem.

Students will be expected to:

4.1 expand binomials of the form $(x+a)^n$, $n \in W$ using the Binomial Theorem
4.2 relate the numerical coefficients in a binomial expansion to the terms of Pascal's Triangle and vice versa.

SEQUENCES AND SERIES

Overview

The non-material world of information processing requires the use of discrete or discontinuous mathematics. Computers are essentially finite, discrete machines, and thus it is essential for students to understand topics from discrete mathematics such as sequences and series to be able to solve problems using computer methods. In studying sequences and series, students should come to understand the power of sequences and series to describe recurrence relations; that is, formulas expressing each term as a function of one or more of the previous terms. Students should understand the role of recurrence formulas for solving enumeration problems since these can be translated easily to computer programs to obtain solutions. Students should use difference-equation techniques to express recurrence relations in closed form; that is, the n^{th} term written as a function of n.

Recurrence relations can be used to model real-world phenomena. For example, the terms in the Fibonacci sequence occur surprisingly frequently in nature and the analysis of these arrangements provides an ideal setting for integrating the study of mathematics and botany.

The study of finite sequences and series leads to consideration of the corresponding infinite cases and concepts associated with limiting processes. Although the study of these concepts is beyond the consideration of this course, an understanding of the concepts contained here will contribute to the meaningful development of the concepts associated with calculus.

Learner Expectations

1. *Students will be expected to* demonstrate an understanding that a sequence is a set of quantities determined by a rule (function) whose domain is the natural numbers and whose range is the terms of the sequence.

   Students will be expected to:

   1.1 recognize finite and infinite sequences
   1.2 write the terms of a sequence given the function that defines it
   1.3 write the terms of a sequence given its recursive definition
   1.4 determine the functions that describes simple sequences.

2. *Students will be expected to* demonstrate an understanding that a series is the sum of the terms of a sequence.

   Students will be expected to:

   2.1 expand a series that is given in sigma notation.

3. *Students will be expected to* demonstrate an understanding that arithmetic sequences are such that each term is equal to the sum of the preceding term and a constant and that an arithmetic series is the indicated sum of the terms of an arithmetic sequence.

   Students will be expected to:

   3.1 apply the general term formula of arithmetic sequences, $t_n = a + (n-1)d$

   3.1.1 solve problems involving the use and application of the general term formula for arithmetic sequences

   3.2 apply the sum formula of arithmetic series, $S_n = \frac{n}{2}(a + t_n)$; $S_n = \frac{n}{2}[2a + (n-1)d]$

   3.2.1 solve problems involving the use and application of the sum formula for arithmetic series
   3.2.2 use technology where applicable.

4. *Students will be expected to* demonstrate an understanding that geometric sequences are such that each term is equal to the product of the preceding term and a constant and that a geometric series is the indicated sum of the terms of a geometric sequence.

   Students will be expected to:

   4.1 apply the general term formula of geometric sequences, $t_n = ar^{n-1}$

   4.1.1 solve problems involving the use and application of the general term formula for geometric sequences

   4.2 apply the sum formula of geometric series, $S_n = \frac{a(r^n - 1)}{r-1}, r \neq 1$; $S_n = \frac{rt_n - a}{r-1}, r \neq 1$

   4.2.1 solve problems involving the use and application of the sum formula for geometric series
   4.2.2 use technology where applicable.

D. BASIC LEARNING RESOURCES

Mathematics 10–20–30

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